CANONICAL CORRELATIONS WITH RESPECT TO A COMPLEX STRUCTURE

BY

STEEN A. ANDERSSON

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OFFICE OF NAVAL RESEARCH

THEODORE W. ANDERSON, PROJECT DIRECTOR

DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY
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1. Introduction

Let E be a vector space of dimension $2p$ over the field of real numbers $\mathbb{R}$. Let $x_1,\ldots,x_N$ $(N \geq 2p)$ be identically distributed independent observations from a normal distribution with mean value 0 and unknown covariance Σ. That is, Σ is a positive definite form on the dual space E^* to E. The maximum likelihood estimator $\hat{\Sigma}$ for Σ is well-known to be given by

$$\hat{\Sigma}\,(x_1,\ldots,x_n) = ((x^*,y^*) \to \frac{1}{N}\sum_{i=1}^{N} x^*(x_i)y^*(x_i) \;;\; x^*,y^* \in E^*) \;.$$

The distribution of $\hat{\Sigma}$ is the Wishart distribution on the set $\wp(E^*)_r$ of positive definite forms on E^* with N degrees of freedom and parameter $\frac{1}{N}\Sigma$. Suppose now that E is also a vector space over the field $\mathbb{C}$ of complex numbers such that the restriction to the subfield of real numbers in $\mathbb{C}$ is the original vector space structure on E. The dimension of E as a vector space over $\mathbb{C}$ is then p. The vector space E^* is then also a vector space over the complex numbers under the definition $zx^* = x^* \circ \bar{z} = (x \to x^*(\bar{z}x) \;;\; x \in E)$, $x^* \in E^*, z \in \mathbb{C}$. The set $\wp_{\mathbb{C}}(E^*)_r = \{\Sigma \in \wp(E^*)_r \,|\, \Sigma(zx^*, y^*) = \Sigma(x^*, \bar{z}\, y^*), \forall\, x^*, y^* \in E^*, \forall\, z \in \mathbb{C}\}$ defines a nulhypothesis in the statistical model described above. The condition $\Sigma(zx^*, y^*) = \Sigma(x^*, \bar{z}\, y^*)$, $\forall\, x^*, y^* \in E^*, \forall\, z \in \mathbb{C}$ is in Andersson [2] called the $\mathbb{C}$-property and in terms of matrices it has the formulation: For every basis $e_1^*,\ldots,e_p^*$ for the complex vector space E^* the matrix for a Σ with the $\mathbb{C}$-property with respect to the basis $e_1^*,\ldots,e_p^*, ie_1^*,\ldots,ie_p^*$ for the real vector space E^* has the form

$$\begin{pmatrix} \Pi & F \\ -F & \Pi \end{pmatrix} \tag{1.1}$$

The statistical problem of testing $\Sigma \in \wp_{\mathbb{C}}(E^*)_r$ versus $\Sigma \in \wp(E^*)_r$ is invariant under the action of the group $GL_{\mathbb{C}}(E)$ of complex one-to-one linear mappings onto the sample and parameter space $\wp(E^*)_r$ given by

$$(1.2) \qquad \begin{aligned} GL_{\mathbb{C}}(E) \times \wp(E^*)_r &\to \wp(E^*)_r \\ (f,\Sigma) &\to \Sigma \circ (f^* \times f^*) \end{aligned}$$

where f^* is the dual mapping to $f \in GL_{\mathbb{C}}(E)$. The restriction of the action to the subset $\wp_{\mathbb{C}}(E^*)_r$ is transitive. Since all tests invariant under (1.2) have a factorization through a maximal invariant function we shall find a representation of a maximal invariant function into $\mathbb{R}_+^p$, describe the distribution as a density with respect to a restriction of the Lebesque measure and state an interpretation of this representation. The matrix for a complex linear mapping of E with respect to a basis of the form $e_1,\dots,e_p$, $ie_1,\dots,ie_p$, where $e_1,\dots,e_p$ is a basis for the complex vector space E is of the form

$$(1.3) \qquad \begin{pmatrix} A & B \\ -B & A \end{pmatrix}$$

The expression $\Sigma \circ (f^* \times f^*)$ from (1.2) in matrix formulation becomes

$$(1.4) \qquad \begin{Bmatrix} A & B \\ -B & A \end{Bmatrix} \begin{Bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}' & \Pi_{22} \end{Bmatrix} \begin{Bmatrix} A' & -B' \\ B' & A' \end{Bmatrix}$$

with respect to the dual basis $e_1^*,\dots,e_p^*$, $ie_1^*,\dots, ie^*_p$ in E^* to $e_1,\dots,e_p,ie_1,\dots,ie_p$ in E .

2. Representation of the maximal invariant

2.1. Lemma. Let Π be a positive definite form on the $\mathbb{R}$-space E. Then there exists a basis $e_1,\ldots,e_p$ for the $\mathbb{C}$-space F such that the $2p \times 2p$ real matrix for Π with respect to $e_1,\ldots,e_p$, $ie_1,\ldots,ie_p$ has the form

$$\begin{Bmatrix} I & D_\lambda \\ D_\lambda & I \end{Bmatrix} \tag{2.1}$$

where I is the $p \times p$ identity matrix and

$$D_\lambda = \mathrm{diag}(\lambda_1,\ldots,\lambda_p) \quad \text{with} \quad 1 > \lambda_1 \geq \cdots \geq \lambda_p \geq 0 \ . \tag{2.2}$$

Furthermore, the matrix D_λ is uniquely determined by Π ; and if $\lambda_1 > \lambda_2 > \cdots > \lambda_p > 0$, then Π also determines the basis $e_1,\ldots,e_p$ uniquely up to the sign of each basis vector.

Proof: Let $e_1',\ldots,e_p'$ be a basis for the $\mathbb{C}$-space E and let

$$\begin{Bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}' & \Pi_{22} \end{Bmatrix}$$

be the $2p \times 2p$ real matrix for Π with respect to $e_1',\ldots,e_p'$, $ie_1',\ldots,ie_p'$. The assertion is then that there exists a nonsingular complex $p \times p$ matrix $Z_1 = A + iB$ such that

$$\begin{Bmatrix} A' & B' \\ -B' & A' \end{Bmatrix} \begin{Bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}' & \Pi_{22} \end{Bmatrix} \begin{Bmatrix} A & -B \\ B & A \end{Bmatrix} = \begin{Bmatrix} I & D_\lambda \\ D_\lambda & I \end{Bmatrix} \tag{2.3}$$

and that D_λ is unique; and in the case where $\lambda_1 > \ldots > \lambda_p > 0$, the columns of Z are unique up to multiplication with ± 1.

The equation (2.3) is equivalent to the complex matrix equations

$$\text{(2.4)}\qquad \begin{aligned} \bar{Z}_1'\left(\tfrac{1}{2}(\Pi_{11}+\Pi_{22})+i\tfrac{1}{2}(\Pi_{12}'-\Pi_{12})\right)Z_1 &= I \\ Z_1'\left(\tfrac{1}{2}(\Pi_{12}'+\Pi_{12})+i\tfrac{1}{2}(\Pi_{11}-\Pi_{22})\right)Z &= D_\lambda \end{aligned}$$

If we define $Z = Z_1^{-1}$ and

$$\text{(2.5)}\qquad \begin{aligned} \Phi &= \tfrac{1}{2}(\Pi_{11}+\Pi_{22}) + i\,\tfrac{1}{2}(\Pi_{12}'-\Pi_{12}) \quad , \\ \Psi &= \tfrac{1}{2}(\Pi_{12}'+\Pi_{12}) + i\,\tfrac{1}{2}(\Pi_{11}-\Pi_{22}) \quad , \end{aligned}$$

then (2.4) becomes

$$\text{(2.6)}\qquad \begin{aligned} \Phi &= \bar{Z}'Z \\ \Psi &= Z'D_\lambda Z \end{aligned} \quad .$$

Since Φ respectively Ψ is the matrix for a positive definite hermitian form respectively symmetric form on the $\mathbb{C}$-space E, it follows from [3] that we can find a complex $p\times p$ diagonal matrix D and a complex nonsingular $p\times p$ matrix Y such that

$$\text{(2.7)}\qquad \begin{aligned} \Phi &= \bar{Y}'Y \\ \Psi &= Y'DY \end{aligned} \quad .$$

By permutation we can obtain that the diagonal elements $d_1,\ldots,d_p$ of D have the property $|d_1| \geq |d_2| \geq \ldots \geq |d_p|$. If we then multiply

the ν'th row of Y with $\exp[-i\theta_\nu/2]$, where $d_\nu = |d_\nu|\exp[i\theta_\nu]$, $\nu = 1,\ldots,p$, and call this new matrix for Z, we obtain (2.6) with $\lambda_\nu = |d_\nu|$, $\nu = 1,\ldots,p$. Since Π is positive definite, we have $1 > \lambda_1 > \ldots \geq \lambda_p \geq 0$. The uniqueness follows from a rather elementary examination of the proof in [3] or from direct matrix calculation. Since every matrix of the form (2.1) with $1 > \lambda_1 \geq \ldots \geq \lambda_p \geq 0$ is positive definite it follows from Lemma (2.1) that the mapping from $\wp(E^*)_r$ onto $\Omega = \{(\lambda_1\ldots,\lambda_p) \in \mathbb{R}^p_+ \mid 1 > \lambda_1 \geq \ldots \geq \lambda_p \geq 0\}$ determined from Lemma 2.1 is a maximal invariant function.

3. Canonical correlations with respect to a complex structure. Interpretation.

It follows from Lemma 2.1 that there exists a basis $e_1,\ldots,e_p$ for the $\mathbb{C}$-space E such that the $2p \times 2p$ matrix for Σ with respect to $e^*_1,\ldots,e^*_p$, $ie^*_1,\ldots,ie^*_p$ has the form (2.1). In (2.1) D_λ is unique; and if $\lambda_1 > \ldots > \lambda_p > 0$, the basis $e^*_1,\ldots,e^*_p$ for the $\mathbb{C}$-space E^* is unique up to a sign for each element. <u>λ_j is called the j-th theoretical canonical correlation of Σ with respect to the complex structure</u>, and <u>e^*_j is called the j-th theoretical canonical linear form of Σ with respect to the complex structure $j = 1,\ldots,p$</u>. Let $x \in E^*$ have coordinates $(\alpha_1,\ldots,\alpha_p, \beta_1,\ldots,\beta_p)$ with respect to $e^*_1,\ldots,e^*_p$, $ie^*_1,\ldots,ie^*_p$. Then

$$\Sigma(x^*,x^*) = \sum_i \alpha_i^2 + \sum_i \beta_i^2 + 2\sum_i \lambda_i\alpha_i\beta_i \tag{3.1}$$

$$\Sigma(ix^*,ix^*) = \sum_i \alpha_i^2 + \sum_i \beta_i^2 - 2\sum_i \lambda_i\alpha_i\beta_i \tag{3.2}$$

$$(3.3) \qquad \Sigma(x^*, ix^*) = \sum_i \lambda_i(\alpha_i^2 - \beta_i^2) \quad .$$

Consider the problem of maximizing $\Sigma(x^*, ix^*)$ under the conditions $\Sigma(x^*,x^*) = \Sigma(ix^*,ix^*) = 1$. This is equivalent to maximizing

$$(3.4) \qquad \Sigma \lambda_i(\alpha_i^2 - \beta_i^2)$$

subject to the conditions

$$(3.5) \qquad \sum_i \alpha_i^2 + \sum_i \beta_i^2 = 1 \quad \text{and} \quad \sum_i \lambda_i \alpha_i \beta_i = 0 \quad .$$

If we suppose that $\lambda_1 > \lambda_2 > \dots > \lambda_p > 0$, we get by using Lagrange's multipliers that the maximum point is achieved at $\alpha_1 = \pm 1$, $\alpha_2 = \dots = \alpha_p = \beta_1 = \dots = \beta_p = 0$, and the maximum value is λ_1. By induction it follows that $\pm e_j^*$ are the only linear forms uncorrelated with $e_1^*, \dots, e_{j-1}^*$ for which $\Sigma(e_j^*, e_j^*) = \Sigma(ie_j^*, ie_j^*) = 1$ and $\Sigma(e_j^*, ie_j^*)$ is maximal. The maximum values are λ_j, $j = 1, \dots, p$.

The canonical correlations $\lambda_1, \dots, \lambda_p$ with respect to the complex structure can be found as the positive roots of the equation

$$(3.6) \qquad \left| \begin{pmatrix} \Sigma_{12}' + \Sigma_{12} & \Sigma_{22} - \Sigma_{11} \\ \Sigma_{22} - \Sigma_{11} & -\Sigma_{12}' - \Sigma_{12} \end{pmatrix} - \lambda \begin{pmatrix} \Sigma_{11} + \Sigma_{22} & \Sigma_{12}' - \Sigma_{12} \\ \Sigma_{12} - \Sigma_{12}' & \Sigma_{11} + \Sigma_{22} \end{pmatrix} \right| = 0$$

where

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}' & \Sigma_{22} \end{pmatrix} \quad ,$$

with respect to a basis of the form $f_1^*, \dots, f_p^*$, $if_1^*, \dots, if_p^*$.

4. The distribution of the empirical canonical correlations with respect to a complex structure.

The estimator $\hat{\Sigma}(x_1,\ldots,x_N)$ for Σ in the observations point $(x_1,\ldots,x_N)$ is given in the introduction. Suppose that $\Sigma \in \mathcal{P}_{\mathbb{C}}(E^*)_r$ and let $e_1^*,\ldots,e_p^*$ be a basis for E^* such that the $2p \times 2p$ matrix for Σ with respect to the basis $e_1^*,\ldots,e_p^*,ie_1^*,\ldots,ie_p^*$ is the $2p \times 2p$ identity matrix. The distribution of $\hat{\Sigma}$ in terms of matrices is a Wishart distribution with a representation as a density with respect to the restriction of the Lebesgue measure to all positive definite $2p \times 2p$ matrices $\mathcal{P}(\mathbb{R}^{2p})_r$ as follows

$$(4.1) \qquad c \cdot |\det \Theta|^{(N-2p-1)/2} \exp\{-\frac{1}{2}\operatorname{tr}(\Theta)\}d\Theta, \ \Theta \in \mathcal{P}(\mathbb{R}^{2p}) \quad .$$

The canonical correlations and linear forms (with respect to the complex structure) of $\hat{\Sigma}(x_1,\ldots,x_N)$ is called the <u>empirical canonical correlations and linear forms with respect to the complex structure.</u> The classical theory of canonical correlations is due to Hotelling [4]. We shall find the distribution of these. If we define Φ and Ψ from the $2p \times 2p$ real matrix Θ , as in formula (2.5) , we have a one-to-one and onto mapping between $\mathcal{P}(\mathbb{R}^{2p})_r$ and $\mathcal{P}(\mathbb{C}^p)_r \times \mathcal{S}(\mathbb{C}^p)$, where $\mathcal{P}(\mathbb{C}^p)_r$ respectively $\mathcal{S}(\mathbb{C}^p)$ denotes the set of positive definite hermitian respectively symmetric $p \times p$ complex matrices, with Jacobian 1. Furthermore, (2.6) defines a one-to-one mapping from $GL_+(\mathbb{C}^p) \times \Omega$ into $\mathcal{P}(\mathbb{C}^p)_r \times \mathcal{S}(\mathbb{C}^p)$, where $GL_+(\mathbb{C}^p)$ is the subset of all nonsingular $p \times p$ complex matrices with a positive real part in the first row and $\Omega = \{(\lambda_1,\ldots,\lambda_p) \in \mathbb{R}^p | 1 > \lambda_1 > \ldots > \lambda_p > 0\}$.

The complementary to the image (which is an open set) of this mapping has Lebesgue measure 0; and therefore from our distribution point of view, we can forget this. To find the Jacobian of this mapping defined by (2.6), we proceed as in [1]. The method is due to Hsu [5]. We have

$$d\Phi = (d\bar{Z}')Z + \bar{Z}'(dZ) \qquad (4.2)$$
$$d\Psi = (dZ')\Lambda Z + Z'(d\Lambda)Z + Z'\Lambda(dZ)\,,$$

and we shall find the absolute value of the determinant of the linear mapping $(dZ,d\Lambda) \to (d\Phi,d\Psi)$ defined by (4.2). This is a composition of

(a) $$\begin{pmatrix} dZ \\ d\Lambda \end{pmatrix} \to \begin{pmatrix} (dZ)Z^{-1} \\ d\Lambda \end{pmatrix} = \begin{pmatrix} dW \\ d\Lambda \end{pmatrix}\,,$$

(b) $$\begin{pmatrix} dW \\ d\Lambda \end{pmatrix} \to \begin{pmatrix} d\bar{W}' + dW \\ dW'\Lambda + d\Lambda + \Lambda dW \end{pmatrix} = \begin{pmatrix} dY \\ dX \end{pmatrix}\,,$$

(c) $$\begin{pmatrix} dY \\ dX \end{pmatrix} \to \begin{pmatrix} \bar{Z}'dYZ \\ Z'dXZ \end{pmatrix} = \begin{pmatrix} d\Phi \\ d\Psi \end{pmatrix}\,.$$

The Jacobians are $|\det Z|^{-2p}$, $|\det Z|^{2(2p+2)}$ respectively $c_1 \Pi_{i=1}^{p} \lambda_i \Pi_{i<j} (\lambda_i^2 - \lambda_j^2)$ for (a), (c) respective (b). Since $tr(\Theta) = 2\, tr(\Phi) = 2\, tr(\bar{Z}'Z)$ and $|\det \Theta| = |\det \bar{Z}'Z|^2 \Pi_{i=1}^{p} (1-\lambda_i^2)$ (4.1) is transformed to the distribution

$$c_2 \cdot |\det \bar{Z}'Z|^{N-p} \qquad (4.3)$$
$$\exp\{-\tfrac{1}{2}\, tr(\bar{Z}'Z)\} \prod_{i=1}^{p} \lambda_i (1-\lambda_i^2)^{(N-2p-1)/2} \prod_{i<j} (\lambda_i^2 - \lambda_j^2)\, dZ \bigotimes_{i=1}^{p} d\lambda_i$$

on $GL_+(\mathbb{C}^p) \times \Omega$. Integrating over $Z \in GL_+(\mathbb{C}^p)$, we get the distribution of $f_1 = \lambda_1^2, \ldots, f_p = \lambda_p^2$:

$$(4.4) \qquad c_3 \prod_{i=1}^{p} (1 - f_i)^{(N-2p-1)/2} \prod_{i<j} (f_i - f_j) df_1, \ldots, df_p$$

on $\Omega = \{(f_1, \ldots, f_p) \in \mathbb{R}^p \mid 1 > f_1 > \ldots > f_p > 0\}$. Formula (13) in [1], p. 324, for $p_1 = p - 1$ and $p_2 = p$ gives the normings constant c_3, namely,

$$(4.5) \qquad c_3 = \pi^{\frac{1}{2}(p-1)} \prod_{i=1}^{p-1} \frac{\Gamma(\frac{1}{2}(N - i))}{\Gamma(\frac{1}{2}(N - p - i))\Gamma(\frac{1}{2}(p - i))\Gamma(\frac{1}{2}(p + 1 - i))} .$$

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Suppose a 2p-variate multivariate normal distribution is of the form of a p-variate complex distribution. The set of such distributions is invariant with respect to a group of linear transformations. The invariants of the set of all 2p-variate distributions with respect to this group are obtained and interpreted. The distribution of the sample invariants is found.

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